RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

M.A./M.Sc. FIRST SEMESTER EXAMINATION, JANUARY 2015

FIRST YEAR

MATHEMATICS

Date : 10/01/2015

Time : 11 am – 1 pm

Paper : I

Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

Answer <u>any four</u> questions : [4×5]

1. State the class equation for a finite group. Show that any group of order p^2 where p is a prime, has nontrivial centre. [2+3]
2. Let G be a non-commutative group of order p^3 where p is prime. Show that the centre of G has order p. Also show that $x^p \in Z(G)$ for all $x \in G$. [2+3]
3. Prove that any abelian group of order 55 is cyclic. [5]
4. Show that any group or order 30 cannot be simple. [5]
5. Define solvable group. Let H be a normal subgroup of a group G such that both H and G / H are solvable. Prove that G is also solvable. [1+4]
6. State the fundamental theorem for finitely generated abelian group. Show that every finitely generated torsion free abelian group is free. Give an example to show that finitely generated condition is necessary. [2+2+1]

Group – B

Answer <u>any six</u> questions : [6×5]

7. Let F be a field. Show that any ring homomorphism of F into F is either trivial or an isomorphism. [5]
8. An ideal I in a commutative ring is called primary ideal if whenever $ab \in I$, either $a \in I$ or $b^k \in I$ for some $k \geq 1$. Show that if I is a primary ideal, then its radical $\sqrt{I}$ is a prime ideal. [5]
9. Show that in a commutative ring with identity, every maximal ideal is prime. Let R and S be two integral domains such that their quotient fields are isomorphic. Is R isomorphic to S? Justify your answer. [3+2]
10. Define irreducible and prime elements in an integral domain. Show that every prime element is irreducible. [2+3]
11. Show that in a principal ideal domain (PID) every non-zero prime ideal is maximal. Show that the result is not true if the ring is not necessarily a PID. [3+2]
12. Let A be a Noetherian ring and ϕ be a ring homomorphism from A onto a ring B. Prove that B is also Noetherian. [5]
13. Show that every Artinian integral domain is a field. [5]
14. Let I and J be two distinct maximal ideals in a ring R with identity. Show that $I \cap J = IJ$. [5]
15. Prove that in the ring $\mathbb{Z}[X]$, the ideal $(5, X^2+2)$ generated by 5 and X^2+2 is maximal. [5]

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